Paper Title: Apprehending Nature Within a Generalized Framework of Complementarity
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Abstract:
Complementarity is the name given to a framework for the analysis of ideas worked out by Niels Bohr as part of his work in the interpretation of quantum theory. Within this framework, it becomes possible to maintain the truth of contrary descriptions, which are considered not as competing contradictions but rather as complementary aspects that are both needed for a complete understanding. Complementarity has been controversial, sometimes hailed as a liberating way to face a complex world but also condemned as obscurantism and a flight from reason. Several thinkers have proposed complementarity as a valuable way in which to view the relationship between science and religion, but this too has been criticized on grounds of logic and has been feared as an excuse to maintain the mutual irrelevance scientific and religious worldviews. Most of these proposals and criticisms, however, have divorced complementarity from its logical development by Bohr in response to contingent facts and within the context of broader epistemological issues. To understand what potential role complementarity may play in the discussion of science/religion issues, we must start by understanding the subtle line of thought devised by Bohr, consisting of four crucial premises: interactions at the microscopic scale are discontinuous (the "quantum postulate"); an observed object cannot be known except through some means of observation; space-time coordination and causality are thus no longer simultaneously compatible; classical concepts like space-time coordination and causality are indispensable for describing physical phenomena. From these premises, the complementarity interpretation of quantum theory can be developed and then expanded to explain both indeterminacy and wave-particle duality. Bohr hoped to broaden the "epistemological lesson" of complementarity to include other sciences, but he restricted its application to empirical studies. In this work, I will develop a set of premises that are analogous to those of Bohr but suitably widened in scope so as to include non-empirical dimensions of existence. For example, the "means of observation" in Bohr's formulation is a non-sentient experimental apparatus, but in the present version this is taken to mean a subjective knower as in the tradition of critical philosophy. By employing a set of appropriate analogies of this type, a generalized framework of complementarity is worked out based closely on Bohr's reasoning but now appropriate for analyses beyond restrictions confined to empirical data. The resulting generalized complementarity framework will then be used to demonstrate that a view of nature based on scientific materialism and a view of nature entailing a sacred aspect are both equally true, do not contradict each other, and are both necessary in order to have a complete understanding of nature. Implications of holding these complementary views of nature as sacred and nature as mundane will be explored, and limitations on each view required for logical consistency will be noted. The usefulness of this methodology for examining traditional problems like creation, design, and the basis of human consciousness will be briefly indicated.

Biography:
Gregory N. Derry is a professor of physics and former chair of the Physics Department at Loyola College in Maryland. He teaches at all levels and maintains an ongoing research program in experimental surface physics. He has also written an introductory book on the nature of scientific inquiry, *What Science Is and How It Works*, published by Princeton University Press. In addition to the chapter on the relationship of science to religion in his book, he has also published an article on the role of mathematics in the religious concepts of various cultures and times, called *Matter, Divinity, and Number*. Designated as a Newman Scholar by his institution several years ago, Prof. Derry presented a critical commentary on Cardinal Newman's concept of science and how it relates to theology and to liberal education. He holds a B.S. degree from Union College and a Ph.D. from the Pennsylvania State University, both in physics. Previous appointments include Lawrence Berkeley Laboratory and the University of North Carolina at Chapel Hill.

Paper:

Introduction

The usual English language meaning of the word "complementary" serves as a good starting point for our discussion. Complementary descriptions of some object or situation are descriptions that capture differing aspects such that the two partial descriptions together form a complete description. Additionally, there may be a connotation that the two partial descriptions are mutually exclusive of each other. If taken in this latter sense, then the two complementary descriptions are contraries that cannot be synthesized into a single unified description but are both needed in order to have a truly exhaustive understanding of the object or situation. We will refer to the premise that such complementary descriptions are sometimes necessary as "complementarity" and note briefly that the premise has sometimes been attacked as logically untenable.[1,2] But complementarity, as so far presented, is somewhat vague. The idea needs to be sharpened considerably before any worthwhile discussion of its logical status is possible.

A more precise and limited use of the word "complementarity" was introduced[3] by Niels Bohr in 1927. The intellectual project that occupied Bohr at the time was devising an interpretation for the newly developed theory of quantum mechanics. A self-consistent mathematical formalism had finally been developed and a good deal of experimental data was in agreement with the predictions of this formalism. However, the physical meaning of the mathematical terms was still unclear. What was this theory actually telling us about how nature operates? No good answer to this question existed, and thus Bohr was searching for an interpretation of the mathematical content of the theory. His search led him to a far-reaching set of epistemological conclusions, revamping the very meaning and purpose of a physical theory. These conclusions were based on an extensive chain of reasoning, and the entire argument was given the name "complementarity" in order to have some brief way to refer to it. Complementarity, in this sense, then refers to a logical

framework for the analysis of ideas; it is a methodological approach rather than an assertion or principle.[4,5,6]

Complementarity has been employed to discuss the relationship between science and religion for over half a century. MacKay[7,8] was an early proponent of this usage, arguing that a scientific interpretation of a given event and a theological interpretation of the same event could neither agree nor conflict with each other (since they are in separate logical categories) but instead represent complementary descriptions of the same event. These issues were further discussed and enlarged by Reich[9], who also concludes that complementarity is a highly valuable approach to relating science and theology. More recently, Watts[10] has offered an extended analysis of the issues that includes consideration of previous criticisms, arguing in the end that science and theology are indeed complementary forms of discourse. The criticisms have come from Barbour,[11,12] Alexander,[1] and Bedau[2] on the grounds that the two complementary descriptions must be of the same logical type, a condition violated in the case of science and theology (or religion more generally). Criticisms have also been offered by Sharpe[13] and by Duce[14] on the grounds that complementarity is an overly limited conception that hinders the attempt to truly engage the two discourses. Finally, it should be mentioned that complementarity has also been used within theological discourse to address issues such as the simultaneous humanity and divinity of Christ.[15,16]

Both the supporters and the critics of using complementarity to discuss the science/religion relationship face a key issue in whether to use complementarity as developed by Bohr and applied in physics or to use complementarity as a generic logical tool independent of any such use in physics. Much of the inspiration for the application of complementarity in the science/religion area certainly stemmed from the prominence and celebrity it attained in a fundamental part of physical science. Yet all of these commentators, both proponents and critics, point out the major difference between complementarities within physics and the proposed complementarities between scientific and religious discourse. The arguments are over the validity of the application of complementarity in the latter case, and there is sometimes a murky ambiguity over whether a given application is construed as the use of Bohr's version of complementarity or the use of a more general conceptualization that has the same name but a differing logical status.

Bohr explicitly intended the complementarity framework he developed to be applied in objective empirical sciences (he had hoped that it would be valuable in many such sciences, especially biology and psychology). Hence, his reasoning must be modified accordingly if we wish to adapt it to problems outside the sciences, as in the science/religion relationship. The purpose of this paper is to generalize Bohr's reasoning process appropriately so that complementarity (in the overall spirit that he conceived it) can be applied validly to realms outside the empirical sciences. The intention here is to produce a methodological tool for epistemological analysis that is more valuable than analogical narratives based on similarities in word usage.

The particular complementarity implied by this development, in the present paper, is also more well-specified than simply "science" and "theology" (or "religion"). Since the chief domain of scientific study is nature ("nature" being taken broadly here to include humans as well as the entire physical universe), and since nature may also be subject to religious interpretations of various sorts, then the common reference of our complementary descriptions is taken to be nature. More particularly, the following claim is made: nature is simultaneously both mundane and sacred. "Mundane" in this context might be thought of as being limited to the presuppositions of scientific materialism, while "sacred" carries an implication of divine immanence and spiritual meaning. Nature is mundane, and nature is sacred. These two contrary claims, within the framework of our generalized complementarity, are both true.

Bohr's Development of Complementarity

One crucial premise of Bohr's thinking is what he called the "quantum postulate." By this he means the discontinuous changes in physical quantities (for example energy) at the microscopic scale. When an atom changes state, for example, the atom's energy changes discontinuously from its initial value to its final value, without passing through any intermediate values. All exchanges of energy in the quantum world share this property, essentially due to the fact that Planck's constant is not zero.

Why is the fact of this discontinuous change important? To understand the importance of the quantum postulate, we must look at the contrasting situation in Newtonian classical mechanics. In classical mechanics, we may add energy to or subtract energy from a system controllably and continuously, letting the amount become arbitrarily small if we wish. In fact, this exchange of energy may asymptotically approach zero. It is just this continuity in classical physics that allows us to precisely define the state of the system. The corresponding discontinuity in quantum physics in turn prevents us from so defining the system's state. Because the ability to precisely define states is central to classical determinism, the implication of the quantum postulate is that such determinism is no longer possible.

The foregoing bold assertion requires more proof, so let's further examine Bohr's reasoning. A second crucial premise is that we only know the properties of a physical system by interacting with it. A totally isolated system has no real meaning for us, because it can disclose no information. Whether such a system even exists is an ontological question, but for all epistemological purposes the system might as well not exist, since we can't know anything about it. In more functional language, the object we wish to study must always be considered as part of a larger system that includes the instrument we use to study it. As a purely practical matter, this statement is just a truism of unknown import; in Bohr's quantum interpretive schema, however, the necessity of including the measuring instrument is elevated to an important philosophical principle. Hence, our knowledge is limited, as a matter of principle, to knowledge gained through interactions with the object we wish to know about. But now combine this premise with the quantum postulate: during the necessary interaction, some uncontrollable amount of energy will be exchanged between the object and the measuring instrument, leaving the

object in an undefined state. Since both the quantum postulate and the interaction are necessary consequences of the act of knowing, then it is legitimate to question whether the object ever had a well-defined state to start with (after all, such a hypothetical state is in principle outside our ability to know).

We do, of course, gain a great deal of information during the interaction between the object and the measuring instrument. In particular, we can specify the location of the object in space at a well-defined time. This ability is important, because specifying the space and time coordinates is absolutely essential to our conventional notions of how to understand physical reality. Predicting how the location of a particle in space varies with time is, in a way, the fundamental aim of classical Newtonian mechanics. Hence, we highly prize retaining the ability to create a space-time description of physical systems, as we do in quantum theory.

The price we pay to retain this ability, however, is the uncontrollable exchange of dynamical quantities like energy and momentum during the interaction. The reason this price is so crucially important is that conservation of dynamical quantities like energy and momentum constitutes the bedrock physical principle upon which we build the entire edifice of our physical understanding of nature. More particularly, these conservation laws are intimately tied to our usual notions of causality. Due to the essential discontinuity in dynamical quantities introduced by the quantum postulate, we no longer have well-defined causal relationships in our description of the system. But causality has been a central concept in the sciences since antiquity, and the heart of Newtonian dynamics is that the application of a force causes a well-defined and well understood change in the motion of a particle. To renounce causality would be to give up any hope of doing science at all. Fortunately, we don't need to renounce causality altogether, because a physical system can be prepared in a perfectly well-defined dynamical state. There is no restriction on our knowing the energy and momentum of a system, from which we can calculate its future behavior using our dependable conservation laws. So, a causal description is perfectly possible. The catch is that in order to do so we must renounce our ability to know the system's space and time coordinates.

Hence we have now concluded that a description of a physical system in terms of space and time is possible, and that a description of the same system in terms of cause and effect is possible, but that these two descriptions are mutually exclusive of each other. And yet, both of these descriptions are absolutely necessary if we are to have a complete understanding of the system. The word that Bohr chose to characterize this situation is *complementarity*. We have two *complementary* descriptions, each of which excludes the other and yet both of which are needed for complete understanding. Bohr summarizes the situation thusly: "On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible, and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there can be no question of causality in the

ordinary sense of the word. The very nature of the quantum theory thus forces us to regard the space-time coordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively."[17]

Bohr's third crucial premise is that in order to describe physical phenomena, we must employ the language and concepts of classical physics and the macroscopic world. This idea always remained the vaguest and most problematic part of Bohr's thinking. In one sense, he meant that our experimental observations ultimately consist of sensory perceptions at a macroscopic scale. The measurement apparatus is always classical, in that sense, even if its operation is based on quantum physics (as most instruments today are). In a related sense, we live in a classical world which shapes our language and our visual imagination; thus we must use these to communicate with each other. But he also seemed to imply that the more abstract concepts of classical physics, like energy and angular momentum, are also indispensable (perhaps because these are only refined versions of our more primitive macroscopic perceptions). In any event, the indispensability of using classical concepts appeared to be self-evidently correct to Bohr. He often stated that it is true, but did not offer very much argumentation to demonstrate this. After all, how can we imagine that which is totally beyond any possible experience we may have? If we grant that this premise is correct, and that we are forced to employ classical concepts to describe microscopic objects, then the complementarity framework follows naturally from the rest of the argument. The quantum postulate introduces an essential discontinuity into our interactions with objects, and such interactions are an essential requirement to know anything about the objects. Space-time descriptions and causal descriptions then of necessity become complementary, and any classical picture that unites such descriptions must perforce be employed also in a complementary way.

The use of complementary descriptions, then, is not merely a convenient choice that we might make or not make as we please. Bohr's argument is that the use of complementary descriptions is quite necessary. It's necessary because it is mandated by the contingent physical fact that the quantum of action (Planck's constant, **h**) is not equal to zero. Due to this fact, we have no choice but to employ the two complementary descriptions (space-time description and causal description). The use of one automatically precludes the use of the other; they are mutually exclusive. And yet, both are needed for a complete description of what there is to know. Moreover, using both (though we can't use them simultaneously) is sufficient to provide a complete description everything that is physically knowable about the system. Note carefully, however, that the quantum postulate (which is a physical fact) has been combined with the necessity for observation, which is an epistemological assertion of principle. Given these two premises, Bohr concludes that complementarity is a necessary feature of how we understand the world.

In contrast, classical physics involved nothing resembling complementarity. Classical physics employs a unified self-consistent description in which all of the variables (space, time, energy, momentum, etc.) simultaneously have precise and well-defined values (recall the last sentence in the passage quoted from Bohr). In fact, it's this very combination of space-time coordination and causality that is the essence of classical

determinism (i.e. the idea that all future events are inevitably and predictably fixed). Newtonian mechanics is deterministic in a way that quantum mechanics is not (since space-time and causal descriptions can't be simultaneously used. But why are we obliged to use complementarity in quantum theory but not in classical theory? The reason is that in classical physics, all quantities are completely continuous; in a sense, Planck's constant *is* equal to zero in classical physics. From a rigorous physical point of view, classical results are merely an approximation, appropriate to the macroscopic world, in which all of the energies are so large that **h** can effectively be ignored (i.e. taken to be zero). In this point of view, quantum physics is a more fundamental theory, which approaches toward the classical results in the appropriate limits. For Bohr, then, the classical world picture, in which the goal was to calculate the deterministic space-time trajectories of particles acted on by causal influences, was merely an outdated framework for understanding nature. He considered complementarity to be a new framework within which to understand nature. Put differently, complementarity is a rational generalization of the old classical picture, a new framework that we can now use to interpret and make sense of the seemingly paradoxical results of quantum theory.

The two most well-known applications of the complementarity framework are to the interpretation of the Heisenberg Uncertainty Principle and to the wave-particle duality of matter and energy. The Uncertainty Principle is a mathematical relationship that quantifies the reciprocal degree of precision with which we can know the position and momentum (there are also several other such pairs of dynamical variables) of a particle. This relationship is a consequence of the mathematical structure of quantum theory, and complementarity offers us a way to understand why it should be so. Wave-particle duality is the mysterious empirical observation that physical entities demonstrate properties of particles under some experimental conditions but demonstrate properties of waves under other experimental conditions. Since waves and particles are conceptually incompatible pictures, this result is paradoxical. The paradox is resolved, however, within the complementarity framework, where logically incompatible views are allowed as long as the conditions of observation for the two views are mutually exclusive (which Bohr and Heisenberg demonstrated is true here).

Generalization of Complementarity

As we've seen, a crucial premise in Bohr's work is the undivided wholeness of the observed system and the means used to observe it (experimental apparatus). An isolated system, not being observed, has no meaning (or at best, an ambiguous meaning). The "observer" in Bohr's interpretation, however, is merely some instrument that records (non-erasably) the result of an experiment; no conscious understanding need be involved. The necessity of including the means of observation in order to give meaning to a phenomenon is at the heart of Bohr's thinking, though, and is integral to the validity of complementarity. Thus, I will analogously make the observer an integral part of the process by which we apprehend nature. In the present generalization, however, this observer *does* need to be an actual conscious human, a knowing subject.

The need for a conscious subject as the observer in our present context seems fairly obvious, given the complementary conceptualizations of nature that we are considering. The justification for including this subject, and the limits this inclusion imposes on the validity of various conclusions we may draw, are important issues. These issues are quite general, and form a central part of the so-called critical philosophy. Going back at least to Immanuel Kant, the subject/object relationship has been a significant epistemological problem, to which Bohr's mentor Harald Hoffding made important contributions. Bohr himself was aware of these implications in his work: "…complementarity is suited to characterize the situation, which bears a deep-going analogy to the general difficulty in the formation of human ideas, inherent in the distinction between subject and object."[18] The ramifications of the necessity to include a knowing subject are central to our analysis here.

In atomic physics, the inseparability of the observer from the object of knowledge carried a drastic implication: knowledge of the object is no longer independent of the conditions of observation. This idea is at the heart of complementarity. Because knowledge of the object depends on the conditions of observation, we must carefully specify these conditions in order to have any meaningful knowledge at all. In atomic physics, of course, the specifications concern experimental arrangements and the desired knowledge is objective knowledge about the system, which Bohr refers to as "unambiguously communicable information."[19] For our problem, knowledge concerning nature will not necessarily be objective. But, the crucial importance of examining and specifying the conditions of observation is once again implied by the inclusion of an observer, just as in Bohr's interpretation of quantum mechanics. How the specification is performed and the scope it includes are considerably different for the case of a knowing subject, extending well beyond the mere description of an experimental arrangement. The kinds of questions being asked; the state of consciousness of the observer; the modes of communication possible and those employed; the role of multiple observers and/or technology used in observation; the effects of culture and history, of time, place, and intention; all these things must be taken into account in order to understand the meaning of any knowledge we may have of nature. To examine more carefully these kinds of conditions in a variety of particular cases, and attempt to draw appropriate conclusions, is an ongoing process (hopefully one that is facilitated by the present work). For now, we simply need to note that the process is necessary. If our analogy holds true, then a proper specification of the conditions under which knowledge of nature is acquired results in the complementarity framework being free of logical contradictions. The conditions under which nature is found to be sacred are not those under which it is mundane; both sets of conditions are valid and hence necessary for a complete view; and the sets of conditions (and knowledge derived from them) must be somehow correlated since they describe, at root, the same world.

We come now to a major premise in Bohr's work for which there is *no* analogy in our generalization, namely the need to employ classical concepts. The reason for this breakdown in the analogical treatment is important and illuminating. The classical concepts (like space-time descriptions, causality, energy, and momentum) needed to be employed because they were intrinsic parts of the integrated worldview evolving

naturally out of our macroscopic perceptions. These concepts are, in this sense, just a natural extension of ordinary language, the very language in which we conceptualize our world to start with. This world is an integrated world (by and large, at least) which hangs together, and the refined classical physics picture based on it very definitely hangs together. Space-time coordination and causality work together seamlessly to produce a complete, coherent, and self-contained worldview. It was this worldview that was shattered by the discovery of the quantum ($\mathbf{h} \neq 0$) and the resulting need for complementarity. In stark contrast, no such coherent and self-contained view of nature in the broadest sense (simultaneously religious, philosophical, scientific, rational, mystical, and empirical) has ever existed. Therefore, no analogy to the limitations on the classical picture imposed by quantum theory and complementarity can exist, because no analogy to the classical picture itself exists. Put a little differently, it's as if we imagined a "wave theory school" and an opposing "particle theory school" as contending factions trying unsuccessfully to achieve a hegemonic picture of macroscopic physical reality, with the impasse finally broken by the introduction of complementarity. Of course, this never happened because both of these (waves, particles) fit coherently into a single overarching picture. Why? Because at the macroscopic level, Planck's constant is so small that we can effectively take it to be zero. If we had lived in a universe where **h** was not small by macroscopic standards, we might imagine the ensuing intellectual strife in the development of physics. In view of the analogy we're building, the apprehension of nature (as mundane or as sacred) has been developed in just such a universe: whatever plays the role of a "quantum of action" in this analogy is not small, and we do indeed suffer from controversy on this question. Hence arises the need for complementarity.

But there is an important part of Bohr's premise that still survives here. Although no analogy to the "classical concepts" exists in the sense just discussed, those concepts that we are in fact using and that collectively make up the "mundane" and "sacred" rubrics (as we explored them earlier) are concepts that are formed under the influence of just those kinds of constraints and conditions stipulated by Bohr. Our concepts must ultimately be limited by our experiences, our language, and the capabilities of our minds. This limitation can't be transcended and we need to use the concepts we have available to formulate our view of nature, even if the analogical "**h**" has turned out to be large for the world we inhabit.

The question of what actually does play the role of **h** in this analogy is difficult to answer. In Bohr's framework, it is the existence of **h** that entails the necessity of joining the observer and observed into an undifferentiated wholeness. In more general epistemological terms, the necessity to include the knowing subject in any consideration of the apprehension of a phenomenon has long been known. For example, Kant wrote that "…if the subject, or even only the subjective constitution of the senses in general, be removed, the whole constitution and all the relations of objects in space and time, nay space and time themselves, would vanish."[20] There is certainly no question about the need for a perceiver in order for there to be a perception, but the influence of this knower on phenomena (and whether anything lies behind the phenomena) has been highly controversial, involving a thick tangle of philosophical issues. Slicing through this tangle is the fundamental question: does how we know the world make any difference to how

the world is? Imparting importance to this question is this fundamental fact: the only world we have is the world that we know. This fact is, perhaps, the best answer possible for the question of what plays the role of **h** in our analogy.

We have now enumerated the main points of a generalized complementarity framework capable of addressing the sacred/mundane antinomy in the apprehension of nature. To summarize these main points: There is an inherent inseparability between the knowing subject and the known world in our apprehension of nature. This is so because we only know the world through our experience of it. This experience is conditioned (in some sense at least) by our minds (broadly considered, including language, culture, neurophysiology, and so on), limiting our available concepts. Due to these limitations, our understanding of nature is only meaningful when the concepts that we use and the conditions under which we use them are carefully examined. The results of such a careful examination yield clusters of concepts (which we can conveniently label "nature as sacred" and "nature as mundane") that appear to be contrary descriptions but that in fact are indicative of differing and mutually non-overlapping conditions of knowing; both descriptions are necessary in order to have an exhaustive description of nature itself.

Some Examples

Let's look at a few specific cases to get a sense of how these ideas might apply to the world. The issues become more pointed when questions of life and consciousness are involved, so we'll start with a deep and powerful instance, namely the manner in which the body of a dead person is considered. "Matter is indeed infinitely and incredibly refined. To any one who has ever looked upon the face of a dead child or parent the mere fact that matter *could* have taken for a time that precious form, ought to make matter sacred ever after. It makes no difference what the *principle* of life may be, material or immaterial, matter at any rate co-operates, lends itself to all life's purposes. That beloved incarnation was among matter's possibilities."[21] These stunning words of William James point directly to the sacred dimension inherent in the world. Within its own context, I find it hard to imagine contradicting this statement. And yet, the believer in a mundane world has plenty of valid arguments to mount: These atoms, making up the body, are no different than any other atoms. The chemical bonds are identical, and subsequent to the body's decay these atoms may well drift into inorganic and less-sacred-seeming forms. Aren't we simply mistaking our own feelings for reality here?

In accordance with the analysis presented in the last section, we need to look closely at the "conditions of observation" in each case to judge the validity of the two views. Specifying these conditions first entails specifying the kind of knowledge we require. Knowledge concerning the person who is now dead, knowledge of this person's life and individuality, is of a different order than knowledge of the dead body's anatomy, of its chemical composition, of its imminent decay. The first kind of knowledge may confer a sacred quality to the dead body, the second kind of knowledge may not. Also, whether there is a strong existential relationship between the "observer" and the dead body can make a profound difference here. That this is so brings us to one of the major issues at stake here: how can the presence or absence of such a relationship make any possible

difference to the status of something in the world? Doesn't the "object" have its own independent existence with its own properties? It appears that we are drifting toward a subjectivistic, idealist, and solipsistic world-view, to which the partisan of mundane existence will object; the body is truly no more than its chemical composition and unbinding atoms, and this sacred quality you perceive is no more than your own emotions within your own mind, not a quality inhering in the thing itself. But this is the very epistemological point that is the crux of my entire argument, because any qualities inhering in the thing itself are *a priori* unavailable to us. We can only know what is available to our senses, our reason, our minds. The body construed as a collection of elements temporarily found in a special configuration is a construction within our minds, no more and no less than the body as the sacred remains of a loved one. I am not promoting solipsistic idealism here. The body is real and possesses qualities, but the only qualities it can posses for us are those it possesses by virtue of its interaction with us. We do not need to deny the mundane dimensions of the properties of the body in order to assert its sacred dimensions. Both are true, hence neither is exhaustive, and no contradiction exists between them (consistent with our examination of the "conditions of observation"). Our differing apprehensions of the meaning of the dead body are complementary.

I admit that I've left out one important point, the trump card of the believer in a mundane world. The view of the body as a collection of chemical elements is an objective view, i.e. multiple observers can all agree on this same chemical composition. In contrast, the sacred qualities of the body may or may not be shared by different persons, and always has in some sense an intensely personal aspect. The objectivity of the mundane view is taken by its supporters to prove its superiority and even its truth. A more formal treatment of this issue will be given in the next section, but note briefly here that objectivity entails losses in addition to its acknowledged virtues, and that our attitudes toward objectivity are part of our cultural history. In an extended discussion of the role of the body (both dead and alive) during the profound cultural shifts accompanying modernism, Romanyshyn expands on these points: "Within the linear, and homogeneous, space of explanation, within that grid where all space has become equal and the same, the heterogeneous pantomimic body has no place. It is a body, therefore, which we no longer need, a body which has become an obstacle; a body for which there is no place is a body ready to be abandoned. It is also, on the other side of this abandonment, a body ready to be reinvented. The corpse is the most visible image of the abandoned body. It is what the human body becomes in our increasing distance from it. It is what the pantomimic, e-motional body becomes for a spectator self behind a window with a heady vision fixed upon an infinite horizon."[22]

Our second example is closer to what one usually associates with "nature" as the object in question. Many places or natural formations have some sacred meaning within a cultural context, and these are not limited to animistic projections and primitive cultures. Consider these comments from a sophisticated Japanese practitioner of Zen Buddhism: "The Japanese love of Nature, I often think, owes much to the presence of Mount Fuji in the middle part of the main island of Japan. Whenever I pass by the foot of the mountain as a passenger on the Tokaido railway line, I never fail to have a good view of it, weather

permitting, and to admire its beautiful formation, always covered with spotless snow and 'rising skyward like a white upturned folding fan,' as it was once described by a poet [Ishikawa Jozan] of the Tokugawa period. The feeling it awakens does not seem to be all aesthetic in the line of the artistically beautiful. There is something about it spiritually pure and enhancing. …. Fuji is now thoroughly identified with Japan. Whenever Japan is talked or written about, Fuji is inevitably mentioned. Justifiably so, because even the Land of the Rising Sun would surely lose much of her beauty if the sacred mountain were erased from the map. … In the beginning, probably, the Japanese were naively attracted to the beautiful which they saw about them; it is possible that they regarded all things in Nature as uniformly animated with life, after the manner of primitive people who look upon even nonsentient things from their animistic point of view. But as they cultivated themselves in the Zen teaching, their aesthetic and religious sensitiveness was further nourished. And this nourishment came in the form of an exalted moral discipline and a highly spiritual intuition. That is to say, the snow-crowned peak of Fuji is now seen as rising from the background of Emptyness…"[23] But Mount Fuji is merely a geological formation, resulting from volcanic forces understood within the context of plate tectonic theory. The snow capping the mountain is a meteorological phenomenon, a vapor-solid phase transition like any other snow. Do these seemingly undeniable facts negate the sacred quality so eloquently described by Suzuki?

Those words were written by a Zen scholar, so Mount Fuji is observed through the eyes (and mind) of a Zen disciple. In Zen *satori* is found a meaning that "is not something added from the outside. It is in being itself, in becoming itself, in living itself."[24] This direct and unfiltered experience of Mount Fuji is different from the experience of the mountain that results from a normal state of consciousness. The resulting meanings and forms of knowledge from the two states are correspondingly different. A direct experience and intuitive knowing cannot yield knowledge of geological history or thermodynamic transformation. Conversely, an analytical consideration of temperature, composition, humidity, and other quantitative data cannot yield the realization of sublime beauty. These two vastly differing states of consciousness thus constitute the "conditions of observation" for the complementary views of nature in this example. Having identified them, we easily see why the two views don't directly contradict each other, since the two states of consciousness won't simultaneously coexist (direct experience and reflective analysis must surely exclude each other). Of course, it still remains to demonstrate that each of these realizations of Mount Fuji is valid and correct, but it's worth noting in this connection that each view has the support of an entire cultural infrastructure and an extensive literature over many centuries.

The last example is also from nature, but not a culturally important object with iconic status for a large group of people. Instead, let's consider a humble piece of crystalline mineral, such as quartz, citrine, or garnet. Crystals are aesthetically beautiful in both sacred and mundane perspectives. Even as mundane objects, crystals hold a tremendous amount of interest. The atoms making up a crystal form a regular periodic array (called a lattice) that has many fascinating mathematical properties. The crystal lattice has a variety of important symmetries (described by the branch of mathematics called Group Theory), which manifest themselves macroscopically in the exquisite facetted shapes of

the crystal. The symmetries of a crystal are also responsible for our ability to understand and calculate its physical properties. As just one characteristic and important example, the symmetries of the crystal lattice produce the so-called "band structure" of its electron states, which is the ultimate basis for all microelectronics technology. For many people, however, the interest of crystals stems not from their mathematical structure but from the sense of mystery and power they possess. Crystals, for such people, are sacred objects. Shamans employed them as such millennia ago, and still today there are people who do so. Is there any validity in such a notion, and can it be consistent with our highly developed scientific understanding of crystalline properties?

I think that upon investigation that we'll find the answer is yes to both questions, but I'll start by pointing out that for me the mundane view is clearly unquestionable. I have personally investigated experimentally the properties of crystals for almost three decades, and never cease to find their physical and mathematical properties fascinating. What are the "conditions of observation" that characterize this endeavor? The key condition here is that we restrict our range of interest to consider only properties that are numerically quantifiable (e.g. positions of atoms in the lattice) and hence measurable. This seems on its face like a hugely restrictive condition, and it is indeed extraordinary that such a fruitful and profound representation emerges from such a seemingly barren starting point. For all its virtues, however, this view does exclude any kind of sacred quality to the crystal. And yet, such a sacred quality does not contradict anything in this view, there is just no room for it. When I "look" at the crystal under these conditions, I can "see" only what I allow. We need now to consider whether there is some other alternative "conditions of observation" that we can define so as to make room for a complementary view.

Such an alternative is offered by phenomenology, a philosophical methodology that "puts essences back into existence…[and]…tries to give a direct description of our experience as it is…"[25] When I "look" at the crystal under conditions set by a phenomenological approach I may "see" an entirely different aspect of the existence of this crystal. "A real being has more materiality, and more significance, than I intend to perceive; when I perceive the real, I must adjust my general intentions to the apparitions which emerge from yonder depth….A real being shows me unsuspected meaning in every perception aimed toward its density; from yonder depth emerges significance beyond the meaning which I had in mind. Real materiality progressively releases hidden meaning during our mental exploration."[26] If we apprehend a crystal employing "conditions of observation" set by a phenomenological approach, we may apprehend emerging from its "yonder depth" a significance that we can justifiably label sacred, a significance not found in our previous encounter with the crystal. Now, our crystal is no less sacred when we encounter it within the context of our mundane world; it's sacred quality was merely less apparent under the conditions we then encountered it. Nor are its mathematical and physical properties any less a part of the total reality of the crystal when it's encountered as a sacred object; once again, these properties are merely less apparent then. The two different apprehensions of the properties of this crystal are complementary: they are mutually exclusive, both are needed for a complete understanding of the crystal, and they do not contradict each other because the conditions of observation needed for each do not

overlap. These conditions, as described here, are quantitative and analytical on the one hand and a phenomenological methodology on the other hand. Yet, it may be misleading to say that these complementary views don't overlap, because after all it is the same crystal. This crystal combines within its totality both of these views of itself, and we who encounter it ultimately do the same.

Complementarity and Metaphysics

I have so far presented arguments for the proposition that humans need to experience nature both as sacred and as mundane, that an inherently religious dimension of reality is on the one hand necessary and on the other hand does not contradict scientific materialism (itself also necessary) within the context of complementarity. If one accepts these arguments as persuasive, then two important and interrelated issues still remain to be resolved: The first question is whether these two complementary faces of reality are attributes of reality itself, or whether they are merely descriptions that we ourselves impose on reality. In other words, is complementarity saying something about the world or something about our psyche? The second question concerns the role and interpretation of objectivity within the context of our complementary views of nature. Should objectivity claim a privileged status in our evaluations of the complementary views, and if so isn't there a bias in favor of scientific materialism? The second question clearly relates back to the first, because such a bias would suggest that the materialistic view stems from a genuine attribute of reality while the sacred qualities of nature are merely our own subjective projections. The questions are fundamental and difficult.

According to Kant, our sources of knowledge can be divided into the content upon which knowledge is based, supplied by the outside world, and the forms that act upon this content, forms that are inherently imposed by the human subjective knower. These forms include the forms of perception (such as space and time) and the forms of understanding (such as causality and quantity). Of the content itself we can know nothing, because all that we know is by virtue of the forms acting on the content; hence Kant postulates an absolute reality consisting of the famous things-in-themselves (noumena) that are forever unknowable. Our known world consists instead of phenomena. The critical philosophy of Kant is thus a nuanced balancing act between the central role of the subjective knower and the objectivity imposed by the existence of an absolute externally real world. Bohr's philosophical mentor, Harald Hoffding, made deeply important contributions to the critical philosophy initiated by Kant, examining both the role of cognition that undergirds the theory of knowledge and the implications of this theory of knowledge for understanding the ontological foundations of Being.

A key point for Hoffding is the role of continuity in the formation of understanding and knowledge. Continuity refers here both to psychological conditions (memory, experience, personality) and to characteristics of the phenomenal world (space, time, causality). For Hoffding, the idea of continuity and discontinuity formed the basis for a unified treatment of cognition, knowledge, and metaphysics. Our very ability to formulate any coherent sense of reality at all depends, according to Hoffding, on the existence of these continuities, and he states that Kant "was not mistaken in declaring that the *demand for*

unity and continuity lies at the bottom of all the forms through which we win or expect to win understanding. He himself has shown that all his categories can be traced back to the concept of continuity…..Logical principles, the principle of causality, and the fundamental doctrines of natural science, all hinge on this concept…"[27] In the broadest sense, this concept of continuity emerges from the fact that nothing is ever known in isolation but only in its relationships to all else. Those cognitional elements characterized by continuity then find commensurate aspects of Being that also possess such qualities of continuity, and from this process issues forth valid understanding. "What appears as an hypothesis from the purely empirical view, becomes, epistemologically considered, a principle, a regulative thought, under whose leadership consciousness may satisfy in the empirical world its demand for continuity and union…..The idea of a working hypothesis points in two directions: on the one hand, as already demonstrated, back to the nature of the thinking consciousness…on the other, to the reality to which the phenomena to be understood belong. A tool must be adapted both to the hand that is to use it and to the object to be worked on."[28] Thus, from the impressions, dreams, fragments, and sensations passing through our minds, the principle of continuity is used to construct and validate reality.

But there is discontinuity as well as continuity. "Even after fruitful principles or working hypotheses have been attained, will Being be completely rendered by them? Or will there always remain an irrational relation *between the principles which may compose our consciousness and the Being itself from which our experiences are derived?* We shall find that under three different forms there is always an irrational remainder, viz. in the relation of quality to quantity, in the significance which the time-relation has for the causal concept, and in the relation between the subject and object."[29] Although each of these three items is of interest, we will focus our attention on the third item, the relation between the subject and object. The reason that the subject/object relationship is of particular interest here is because of the central role that it plays in the development of the generalized complementarity framework we are using, in which the necessity to include a knowing subject is a central premise. We must explore the consequences of this premise.

In Kant's formulation, there is a fairly clean distinction between the subject, who supplies the forms, and the object, which supplies the content. Hoffding notes that this distinction is untenable, because the subject too is part of the world. The forms are not simply given but must be worked out during the process of creating knowledge. "In every cognition we can distinguish between *a subjective* and *an objective element*, between a knower and the thing known; both terms, however, are only given in mutual relation….we nowhere and at no time possess the pure Subject, with its forms, as an antithesis to a pure object….we really set up an objectively determined Subject (S_O) as the reverse of a subjectively determined Object (O_S)."[30] In other words, while our knowledge of the world is conditioned by our forms of perception and understanding, these very forms themselves are conditioned by the world we wish to have knowledge of. Absolute and complete knowledge can never be guaranteed under these conditions. Indeed, the problem is even worse than it initially seems, because the mutual subject/object relationship maintains itself recursively. "Each refers to the other indefinitely, and the

irrational crops out in the fact that an infinite series (of the type: $S_1\{O_1\{S_2\{O_2\{\cdots$) is both possible and necessary…..the springs which feed the stream of thought are inexhaustible"[31]

Before we go too far down this road, we should pause to think about the objections that a common-sense realist might have at this juncture. After all, how can it make any difference to the fundamental workings of the universe whether humans even exist or not, much less how they perceptually interact with it? Surely there must be rules (to be discovered by science) and some ontological ground that are independent of human consciousness. "Humanity is not the center or the measure of the universe or of reality. The universe is not dependent upon human perception….The quest here is for an objective picture of the cosmos as it would exist and function without the alleged contributions of human observers…..In order to have knowledge and intelligibility regarding our transactions with the external world, some form of independent reality or objectivity is essential….physical reality is not partially created when the experiencing observer supplies the categories (as in Kant)….Reality is not constituted by, or contingent upon, human mental or 'observational' activities."[32] We meet here with a so-called "minimum metaphysics" that makes only those assumptions that are needed for clear thinking and seem obviously true. One of these important assumptions is the superiority of objectivity, taken to be so self-evidently correct that it requires no argument in its favor.

The question here, however, is not whether objectivity is valuable (which I believe it is); the question is whether objectivity is unlimited in its scope of validity and application. Hoffding's argument, as we've seen, is that our objective view of reality is constructed out of the elements that are bound by relationships of continuity and causality. He believes that we must always strive to enlarge this realm of objective reality and understanding, but that it will always face limits imposed our own limited apprehension of Being. "…we run up against the irrational, and here perhaps we see most clearly how inexhaustible Being is in comparison with our knowledge….Being may possess attributes that cannot be comprehended or defined by means of the dimensions in which our thoughts can move…..Knowledge, however rich and powerful it may be, is after all only a *part* of Being….An exhaustive concept of reality is not given us to create."[33] The independent existence of the cosmos is not in dispute here. The dispute is concerned over what *we can know* about this independent cosmos. I find Hoffding's argument persuasive, and it follows from this that objectivity constitutes an invaluable but still-limited horizon; other horizons, grounded in apprehensions of other aspects of Being, may also validly exist.

There *is* a manner in which objectivity is privileged, and the foregoing should not mislead us into accepting any assertion as equivalently valid as any other assertion. The formal rules of logic and rationality in combination with their empirical correlates still reign uncontested over their proper domain. Incorrect or nonsensical propositions within this domain should be treated as ruthlessly as if the domain were subject to no limits or restrictions, and this includes propositions that claim some warrant based on revealed text, ecclesiastical authority, or majority opinion. In applying the complementarity framework,

we need to examine carefully whether a proposition falls within this domain or not—what I have previously referred as the conditions under which knowledge is acquired. The argument I have been developing claims that there are legitimate conditions under which knowledge is acquired such that this knowledge does lie outside the domain.

The believer in a mundane world still has two arguments left. The first argument is to claim that their horizon is still the only legitimate horizon, and that any other proposed horizon is illusory. But the only basis I can see for this claim is to deny Hoffding's outcroppings of irrationality and maintain that Being possesses only a continuity congruent with formal logic and rationality, and that the knowing subject has some transcendental status unconditioned by the world (i.e. the subject and object distinctly split). The relationship between the subject and object has already been dealt with in detail, and the assumption of the continuity of Being is difficult to maintain when even the continuity of formal logic itself has been shown to be incomplete. Godel's theorem has unambiguously demonstrated "that it is impossible to establish the internal logical consistency of a very large class of deductive systems—elementary arithmetic, for example—unless one adopts principles of reasoning so complex that their internal consistency is as open to doubt as the systems themselves."[34] If even the formal rules of logic are not self-contained and consistent, what grounds are there to assert that Being possesses these qualities?

The second argument, and perhaps the most difficult to refute, is to decouple epistemology from ontology and contend that what we know or don't know is unrelated to what is. Since the basis of my argument for complementarity has been primarily epistemological, this contention strikes a serious blow against it. On the other hand, what can we possibly *say* about the metaphysical ground of Being except what we *know* about it? Any positive statements that might be made, from the leanest minimum metaphysics to the most florid and grandiose systems, can be no more than dogmatism if they are divorced from our theory of knowledge. Hence, all of our arguments for complementarity on epistemological grounds must and do illuminate our ontological views. The problem of Being and its relation to knowledge was also considered by Hoffding. He points out that since knowledge is always knowledge of relationships between things, then complete knowledge of a totality is inherently self-contradictory. If a totality is to be compared to something else, then it is by definition *not* a totality. Remarking that the problem is equally severe whether we restrict our concept of totality to empirical realms or broaden it to include the transcendental, Hoffding notes that "the antinomy is the same in both cases. The irrational meets us here as it did in the problem of knowledge…..In their different systems of thought, the philosophers have been too sure that Being in itself was a closed and constant totality…."[35] After analyzing the problem in terms of various "type-phenomena" such as life, thought, matter, intelligibility, causality, plurality, and monism, Hoffding concludes that the unifying power found in Being will always be checked and limited by the irrational power found there too, and that the conflict between these opposing tendencies leads to the further development of a Being in the process of Becoming. He then explores the basis of the unifying tendency in Being, carefully considering both of the traditional sources: matter (materialistic metaphysics) and mind (metaphysical idealism). Not only does Hoffding determine that

neither of these solutions is adequate, he also notes that they do not necessarily exhaust all the possibilities. "…the difficulty would remain that matter could no more be derived from the psychical than the psychical from matter…..but there is no proof that there is no other attribute in being besides these two…..The empire of Being may be much vaster than the possibilities of our experience. Here, again, it is true that the world is great, but our mind is small; again we come upon the irrational…..The possibility that there are more forms than our experience exhibits may signify that the whole problem lies deeper than has been supposed."[36] The argument here is that epistemology and ontology are indissolubly linked, and that we indeed can draw some valid metaphysical conclusions based upon our analysis of how and what we know. Based on this reasoning, I would assert that the complementary views of nature as both sacred and mundane, as proposed here, are not merely reflections of our internal mental states but are instead valid descriptions of a reality outside ourselves.

Summary and Prospects

The discussion of issues in the relationship between science and religion in terms of complementarity has historically suffered from several deficiencies. The epistemological analysis of Bohr has either been ignored (in favor of *de novo* logical constructions of a meaning for complementarity) or applied as is (in which case it is not appropriate since it is restricted to use in the empirical sciences). In addition, well-defined statements of the purported complementary descriptions have usually been lacking. This paper has used Bohr's work as a starting point but has suitably generalized the argument so as to make it appropriate for science/religion issues. The generalized complementarity framework is then applied to a more well-defined claim: nature is sacred *and* nature is mundane. A particularly important point that emerges from the generalization of complementarity is that we must examine carefully the conditions under which knowledge of nature is acquired, and this methodological process is illustrated by a set of examples. Remaining questions about how (or whether) the world can be affected by our knowledge of it, and the implications of this for the validity of the complementary descriptions of nature, are addressed in the final part of the paper.

One last question to ask is whether this complementarity framework and its attendant methodology is a productive way to think about science/religion issues. Can we gain any new insights or resolve any old problems by thinking in this way? To explore this question, I am planning to use the complementarity framework to analyze a number of important and longstanding problems in the science/religion area, including cosmogony, human consciousness, and design in nature. The results of this work will be a good indicator of the fruitfulness of complementarity as a tool for thinking about issues that emerge from the relationship of science and religion.

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