

Paper Title: Mathematical Aspects of Freedom and Determination  
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Abstract:

**Freedom** along with personal **responsibility** is one of the main notions of Christian theology. Another issue of importance in Christianity is **determination** or **predestination**. Daring not to say something new about these deep and paramount philosophical questions, we shall give several brief observations concerning similar phenomena in pure mathematics, namely in set theory, beginning 100 years ago.

Biography:

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Paper Text:

**Freedom** along with personal **responsibility** is one of the main notions of Christian theology. Another issue of importance in Christianity is **determination** or **predestination**. Daring not to say something new about these paramount and deepest philosophical questions, we shall give several brief observations concerning similar phenomena in pure mathematics, namely in the set theory, firstly emerged 100 years ago.

In 1904 Ernst Zermelo stated explicitly his famous **axiom of choice** (AC):

**for any disjoint family of non-empty sets there exists a set which has exactly one common element with each of the initially given sets.**

In other words, the AC postulates that if one has several bags with beans then it is possible to fill one new bag by beans taken by one from each of the initial bags. This assertion seems obvious for any family of finite sets in which one may take in each of these sets **the first** element thereof. However in general case of arbitrary infinite family of infinite sets without any prescribed structure one cannot guarantee a constructive method to choose some element in each set. So the Zermelo axiom implies that the elements' choice may be made quite freely, i.e. without any compulsion or motivation whatsoever. Thus AC demonstrates in mathematical language the free will discussed for centuries by philosophers and religious thinkers.

The accepting of AC opens broad ways in obtaining lots of results in many branches of mathematics, e.g. in proving the Hahn-Banach theorem on extension of bounded linear functionals or famous Tikhonov assertion on bicomactness of arbitrary products of bicomacts. However the results obtained by using the AC are totally non-constructive, they affirm the existence of mathematical objects with properties required but yield no idea how to build such objects. Another equivalent forms of AC are so called Zorn lemma and transfinite induction principle (the latter being the broad generalization of the classical mathematical induction principle formulated by Blaise Pascal in 17<sup>th</sup> century).

This paradoxality of the AC is manifested also in many logically repulsing theorems, as say such one: a ball of radius 1 may be divided (of course,

*inconstructively*) into finitely many parts, which after certain motions in the space form TWO balls of the same radius! It looks extremely strange but in fact it is a direct logical consequence of the axiom of (free) choice. Therefore the theorems established **without AC** are regarded as more reliable and authentic. Further research has shown that both the axiom of choice and its negation are consistent with the rest of the Zermelo-Frenkel (ZF) system of the set theory axioms.

The determination is usually associated with oppression, unwillingness and is understood as something completely opposite to freedom. The mystery of the human freedom in view of the Divine omniscience and omnipotence was revealed by the Holy Fathers of the Orthodox Church (4th-14th centuries). In mathematical logic of the end of the 20th century the axiom of determination (AD) was invented in order to eliminate the AC paradoxes and at the same time to save its positive aspects. To explain the situation imagine that the segment between 0 and 1 is divided into two equal halves, each of them being then divided likewise and so on up to infinity. Let further there be two persons (Players) who play the following Game: the Player 1 chooses one of two halves (the right or the left one) of the length  $1/2$ , then the Player 2 chooses one of the two sub-halves (of the length  $1/4$ ) of the subsegment chosen on the step 1, then again the Player 1 chooses the sub-sub-halves and so on up to infinity. The results of these successive choices of the two Players is certain point of the initial segment which is determined uniquely by the strategies of the two Players. Briefly AD is stated as follows: for any set  $S$  of the segment  $[0,1]$  either there exists such a strategy of the Player 1 which guarantees that the resulting point of the Game belongs to the set  $S$ , or there exists such a strategy of the Player 2 which guarantees that the resulting point of the Game does not belong to the set  $S$ . This new axiom helps us to avoid all discrepancies of the AC, in particular, the AD guarantees that any bounded set on the line may be ascribed with certain non-negative measure (what is rejected by the AC). On the other hand, Tikhonov and other similar theorems in topology and functional analysis remain valid under the AD instead of AC.

In theological notions the Game is interpreted as an earthly life of the human being, Player 1 is the Almighty God Who created the Player 2 (i.e. the free person), the set  $S$  stands for the eventual state of a man in the God's design (i.e. salvation for everlasting life), the strategy of the Player 2 is the sequence of person's moral choices, whereas the strategy of the Player 1 are the Lord's commandments and His responses to the human violations thereof which include sorrows, disasters and "all other things that work together for good to them that love God, to them who are the called according to His purpose" (Romans 8, 28). The person's freedom is thus his or her own heart's acceptance or rejection the God's will.

**God, though Almighty, can't save man without man's desire!**