Paper Title: The Role of Mathematics in the Science and Religion Discussion
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Abstract:
With a few notable exceptions the considerable and rapidly growing literature in the science and religion field has seen very little written from the standpoint of mathematics. Yet mathematics is seen as the language of science and has been referred to as the queen of the sciences. Mathematics has also had considerable impact in the development of philosophy, which in turn has been very influential in theology.

This paper is organized around four different quests in the history and development of mathematics: the quests for definition, truth, foundation, and certainty. Reflections on how this might impact theology are included in the discussion of each quest.

The quest for definition focuses on how the concept of number has changed over time in the history of mathematics. From the early Greek ideas of Pythagoras to current understandings, the idea of number has changed as new problems and anomalies developed. While the expansion of the number concept was often controversial the mathematical community did accommodate the new ideas and see how they were consistent with the earlier concepts.

The quest for truth also begins with the Greek understanding that truth about the world is best justified with deductive thinking as exemplified by the geometry of Euclid. This approach was dominate for over two millennia but was challenged by the development of the non-Euclidean geometries of Bolyai, Lobatchevsky, and Riemann in the early 19th century. The questioning of the deductive method, along with the rise of the empirical method in science, led to a significant reappraisal of claims for truth not only in mathematics and the natural sciences, but in the social sciences and humanities as well.

The quest for foundations looks at the impact of Descartes and his method on the understanding of providing firm foundations for knowledge. In mathematics this led to the foundations controversies of the early 20th century pitting the logicist, the formalists, and the intuitionist against each other. The foundations discussion was changed forever with the mathematical results of Kurt Gödel who proved that any mathematical system that included basic arithmetic contained statements that could not be proven in that system. Furthermore, such systems could not be shown to be free from contradictions.

The quest for certainty looks at the centuries old claim that mathematics provides certain knowledge. The claim for certainty was replaced with a search for consistency, which

was then abandoned as unachievable. So where did that leave knowledge in general. If mathematics could not establish certainty could any other system aspire to the same?

For each of the mathematical quests we also look at the implications for the science and theology discussion. Does mathematics, which is seen as the language of science, shed light on the nature of science? Do the concepts of mathematics which had major impact on certain philosophies, which in turn influenced theology, have insight for our current theological discussions? What can mathematics contribute to the science and religion discussion?

Biography:
P. Douglas Kindschi is currently professor of mathematics and philosophy at Grand Valley State University in Grand Rapids, Michigan, where he previously served for over 20 years as the Dean of Science and Mathematics. His interest in the science and religion discussion goes back to his graduate studies at the University of Chicago Divinity School and his leading a campus ministry science-religion program while completing his Ph.D. in mathematics at the University of Wisconsin. At Grand Valley State University he developed the course *Science, Mathematics, and Religion: Ways of Knowing*, which received a Templeton Course Award. He founded and has led for the past seven years a faculty discussion reading group in science and religion. He currently directs a new Local Societies Initiative program which is bringing together an interdisciplinary, inter-institutional, and interfaith dialogue for the greater Grand Rapids area.

Paper Text:
The story is told of the great 18th century German mathematician Leonhard Euler, whose father was a Calvinist minister and had insisted that young Leonhard study theology and Hebrew. Eventually Leonhard convinced his father to let him pursue mathematics. The story, probably apocryphal, has Euler confronting the famous French scholar and atheist Diderot with a spurious mathematical proof for the existence of God. Euler, it is said, accepted the invitation to meet Diderot and when the day arrived, so the story goes, Euler strode up to Diderot and proclaimed:

"Monsieur, $(a + bn)/n = X$, donc Dieu existe; répondez!"

In the past, the French scholar had refuted many clever philosophical arguments for existence of God, but seeing a mathematical formula for which he had no comprehension, he was unable to respond. [Guillen 1983, 1]

I truly doubt that mathematics will come up with a proof of God's existence, yet there is much that can be learned from mathematics as one approaches the science-religion discussion. With a few notable exceptions[1] the considerable and rapidly growing literature in the science and religion field has seen very little written from the standpoint of mathematics. Yet mathematics is seen as the language of science and has been referred to as the queen of the sciences. Mathematics has also had considerable impact in the development of philosophy, which in turn has been very influential in theology.

This paper is organized around four different quests in the history and development of mathematics that could contribute to the science-religion discussion.

QUEST FOR DEFININTION:

While the history of mathematics does not begin with the Greeks, it certainly took a form that would have significant impact on both philosophy and theology. Prior to the Greek mathematics, arithmetic was used for construction, commerce and trade but was not a developed system of thought. It was Pythagoras (6^{th} Century BCE), on the island of Samos, who developed an elaborate combination of mathematics and religion.

Whole numbers had special religious significance. For example:

1 = reason
2 = female
3 = male
4 = justice (squaring of accounts)
5 = marriage
6 = creation

10 was the holiest, and represented the universe, since,

1 represents a point,
2 points determine a line, hence one dimension,
3 points determine a triangle, hence two-dimensional space,
4 points determine a tetrahedron, hence three-dimensional space,
And the sum is 10, representing all of the dimensions and hence the whole universe.

Ratios of whole numbers were also important and seen as providing the basis for the musical scale. For example a string plucked at the one half mark produced a sound one octave above the base, while a string at 2/3rds produced a sound representing a fifth. Music and astronomy could be linked to arithmetic and geometry and the four subjects made up the quadrivium. The Pythagoreans established two important insights that would influence mathematics and science for centuries, even to today. Nature is built upon mathematical principles and number relations reveal an underlying order in nature. [Kline 1980, 15]

Bertrand Russell credits Pythagoras as being "intellectually one of the most important men that ever lived" in that his combination of mathematics and theology set the stage for religious philosophy through the Middle Ages and into modern times. "In Plato, Saint Augustine, Thomas Aquinas, Descartes, Spinoza and Kant there is an intimate blending of religion and reasoning…. The whole conception of an eternal world, revealed to the intellect but not to the senses, is derived from him." [Russell 1945, 31, 37]

For the early Greeks, the natural numbers and the ratios of natural numbers were the essence of mathematics and, in fact, the basis for knowledge of the real world.

Pythagoras is also credited for the theorem which relates the three sides of a right triangle, namely, $a^2 + b^2 = c^2$. This relationship, however, created a significant problem for the classical Greeks. A very simple triangle with two legs each equal to 1 would yield a hypotenuse with a length whose square would be 2. But the Greeks also proved that there was no such a number of the form n/m where n and m are both natural numbers. Such "irrational" numbers were not acceptable to the Greek mathematicians, and even as late as the 17th century there were mathematicians reluctant to accept such numbers.

But for many mathematicians negative numbers were an even greater problem. Mathematician, theologian and close friend of Pascal, Antoine Arnauld (1612-1694) argued that negative numbers could not exist since -1 would be less that 1 and so the proportion -1:1 = 1: -1 would mean that a smaller number is to a greater number as the greater number is to the smaller. [Kline 1980, 115]

Others argued that if negative numbers existed they would have to be bigger than infinity since when dividing 1 by x we get progressively larger numbers, as the denominator gets closer to zero. Hence dividing by an even smaller number, namely a negative one, would yield an even larger number, one greater that infinity. Negative numbers were actually more bothersome than the irrationals since they did not have a geometrical interpretation that the irrational numbers did.

The battle over irrational and negative numbers were minor scuffles compared to the debates over complex numbers and the expression $\boldsymbol{i} = \sqrt{-1}$. In the 1700s mathematicians of the stature of Leibniz, Euler and John Bernoulli debated the meaning, if any, of complex numbers. Euler, for example, in his *Algebra*, the best text in the 18th century says:

> The square roots of negative numbers are neither zero, nor less than zero, nor greater than zero. Then it is clear that the square roots of negative numbers cannot be included among the possible numbers. Consequently we must say that these are impossible numbers. And this circumstance leads us to the concept of such numbers, which by their nature are impossible, and ordinarily are called imaginary or fancied numbers, because they exist only in the imagination. [Kline 1980, 121]

The history of accepting new concepts of number can be summarized by just reviewing the terms used for the new entries proposed for inclusion in the concept of number. We have "natural numbers, rational numbers, real numbers" contrasted with the "negative numbers, irrational numbers, and imaginary numbers."

Eventually, however, mathematicians found appropriate definitions and rules of manipulation so all such numbers could operate in the familiar patterns that had been accepted for the natural and rational numbers. Rules that we now call associative, commutative, and distributive would apply for all of these numbers. In the field of the complex numbers the other number systems such as the real numbers, rational numbers, and integers were all subsets in which the rules for complex numbers were consistent with the rules that had developed historically for these more "normal" numbers.

The story got a bit more complicated, however, with William Hamilton's invention of the quaternions and Caley's development of matrix algebra in the 1800s. In both cases we now have algebras in which the familiar commutative law did not hold. That is, ab = ba does not hold for all "numbers" in the quaternion and matrix algebras systems.

Later in the 19th century another controversy emerged over Cantor's development of an algebra for transfinite numbers where even the intuitive understandings of smaller vs. larger sets did not hold. When the set of even natural numbers was the same size as the set of all natural numbers, let alone the same size as the set of all rational numbers these concepts were challenged as stretching much too far our concept of number. As each new crisis emerged, these new definitions actually led to an enhanced concept of number leading to a more interesting and even more useful arithmetic.

Could there be a lesson here for other fields of study? If even the precision in a field such as mathematics experiences new definitions and expansions of concepts such as number, perhaps in our theology we need to be open to new definition that may not conform to the previous understanding. We need to be open to new opportunities to expand our concepts which may bring us even richer understanding. For Christians, the transition from the Hebrew Scripture understanding of God to the New Testament revelation enhanced the understanding of God while at the same time providing a new a deeper appreciation of the God of the Hebrews. Interfaith dialogue can enrich and expand the ideas of each party and lead to richer understanding. When we limit our definitions only to the terms that we find in our current comfort zones, we foreclose the opportunity to develop new and deeper concepts in our religious understanding.

In today's world the discovery of multiple galaxies, black holes, dark matter and dark energy expands greatly our understanding of the extent and nature of the creation and potentially our understanding of how God works in this universe. Our deeper understanding of quantum mechanics, DNA, genetic code, and evolutionary development can lead us to new and deeper understanding of how God works in the natural processes.

Could such ideas as *panentheism* and the process theology, building on Whitehead's philosophy, lead us to a deeper understanding of God and the relationship of the divine to the natural order? A new appreciation of world religions and the various concepts of God expressed through different traditions can also expand our own understandings and give us a deeper understanding of that which is infinitely greater than us.

It is no wonder then that the Bible and other religious texts use so much metaphor when talking about God. If even in mathematics the quest for definition has proven so difficult, yet has also led to deeper understanding, so perhaps also in religion and theology we must be open to new definitions as well as new ideas and concepts.

QUEST FOR TRUTH:

For Pythagoras, as well as for Plato and Euclid, mathematics was a description of ultimate reality. The knowledge obtained from mathematics was more to be trusted than any empirical knowledge; it was "eternal knowledge." Plato in the Republic states it clearly: "The knowledge at which geometry aims is knowledge of the eternal, and not of the perishing and transient" [Republic, Book VII]

This concept of the cosmos operated by mathematical laws, which could be understood by the human mind, became a foundation for the development of science. For over two thousand years it was accepted that mathematics was in fact a description of reality. Mathematics was the ideal science, the "queen of the sciences," the most basic and the most trustworthy of the sciences.

The ideal model for such a science was available in Euclid's geometry. Beginning with self-evident truths and using the deductive method a significant body of knowledge about the universe was established. This was seen as factual knowledge about the real world, knowledge that was in fact better and more secure than any knowledge from the senses, which could be interpreted differently, could change depending on the observer, and could also change over time. Geometry never changed, there were never different interpretations and once something was proven by such method is was always true, never subject to revision with the discovery of new evidence.

Euclid, who lived in Alexandria about 300 B.C.E., set forth the basic axioms and postulates upon which his geometry would be developed. For the most part they were quite self-evident:

Axioms:

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Postulates:

1. It is possible to draw a straight line from any point to any point.
2. It is possible to extend a finite straight line continuously in a straight line.
3. It is possible to describe a circle with any center and distance (radius).
4. All right angles are equal to one another.
5. That if a straight line falling on two straight lines makes the interior angles on the same side less that two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

The axioms and first four postulates did seem quite clear, but there was concern about the fifth postulate. Not only was it not as self-evident it was even hard to understand without drawing a picture. Furthermore it made a claim about something to be extended "indefinitely." Euclid must have shared this concern since he avoided using it in any of the proofs until he had established all that he could without using the fifth postulate.

Over the centuries various attempts were made to prove it from the other axioms or to find alternate forms that seemed more self-evident. In 1795, John Playfair showed that this fifth postulate of Euclid is equivalent to the following:

> Through a given point, P not on line ***l***, there is one and only one line in the plane of P and ***l*** which does not meet ***l***.

This seemed easier to understand and became the common way for geometry to be taught. In this form it became known as the "parallel postulate or axiom." It still made a claim about a line that was "extended indefinitely."

Another attempt to prove the parallel axiom was to deny it and thereby try to get a contradiction. Some mathematicians in the 1700's actually did develop strange results, which made them believe that the Euclidean postulate must be correct, but they didn't actually produce contradictions. Finally 100 years later, mathematicians, Gauss (1777-1855) in Germany, Bolyai (1802-1860) in Hungry and Lobatchevsky (1793-1856) in Russia did produce geometries that assumed that there was more than one line through a given point parallel to a given line. There were some strange results, such as triangles having less that 180 degrees, but the geometry did prove to be consistent.

The effort to work out this alternate geometry so consumed Janos Bolyai that his mathematician father wrote to him as follows: "For God's sake, please give it up. Fear it no less than the sensual passion, because it, too, may take up all your time and deprive you of your health, peace of mind and happiness in life." [Quoted in Davis and Hersh 1981, 220-1]

Later, Riemann (1826—1866), a student of Gauss, developed a geometry in which lines were not infinite in length and there were no lines parallel to a given line through a given point. This also produced an apparently consistent geometry where many theorems could be proved, including the theorem that a triangle had more than 180 degrees.

What for more that 2,000 years had been had been the absolute truth about space, namely Euclidean geometry, was now seen as one of a number of geometries any one of which could be the actual geometry of physical space.

In the twentieth century "non-Euclideanism" spread into other fields and, along with the pragmatism of James and Peirce, became a critique of the realist positions in philosophy and the social sciences. The philosopher Morris Cohen pointed to the impact of the new geometries by stating: "The Kantians, however, are wrong in claiming absolute logical necessity for material principles such as those of Euclid's geometry, Newton's mechanics or Christian ethics." [Cohen 1918, 688]

Edward Purcell in tracing the development of democratic theory in his classic, *The Crisis of Democratic Theory: Scientific Naturalism & the Problem of Value,* devotes a whole chapter to "Non-Euclideanism."

> If non-Euclidean geometry proved that there were alternate but logically unassailable systems of geometry based on axioms fundamentally different from those of Euclid, then it appeared obvious that deductive reasoning, by assuming contradictory postulates, could produce systems of ethics radically different from that of traditional Christianity. …The concept of non-Euclideanism, generalized to include all types of deductive thought, robed every rational system of any claim to be in any sense true, except insofar as it could be proved empirically to describe what actually existed." [Purcell 1973, 53]

Mathematicians were also not reluctant to join the chorus of extolling the impact of the non-Euclidean geometries not only for mathematics but also for all of human thought. Eric Temple Bell, president of the Mathematical Association of America and member of the National Academy of Science argued in his book, *The Search for Truth*, that there was no such thing as truth. Of the development of the non-Euclidean geometries in 1826, Bell declared it to be "…the most important advance our race has ever made in its attempt to understand its reasoning processes." [Bell 1934, 22]

Morris Kline extended the sentiment further: "The importance of non-Euclidean geometry in the general history of thought cannot be exaggerated. Like Copernicus' heliocentric theory, Newton's law of gravitation, and Darwin's theory of evolution, non-Euclidean geometry has radically affected science, philosophy, and religion. It is fair to say that no more cataclysmic event has ever taken place in the history of all thought." [Kline 1953, 428]

With the development and acceptance in the mathematical community of these alternative geometries we now see more than one model and the correct one to use depends on the setting and other empirical factors. Mathematics builds the models but cannot say which is to be used in which context in the real world.

Mathematics had to give up the Greek ideal of finding Truth through the deductive methods of Euclid. The development of non-Euclidean mathematics showed us that there are multiple models for geometry, each of which is consistent and in fact each of which can be seen as having a model for which they apply even in Euclidean space, i.e. the sphere for Riemann and the pseudosphere for Lobachevski and the others.

If mathematics is not the desired method for discovering absolute truth, what then is to take its place? Early in the 20th century the logical positivists claimed that only that which is empirically verified can be counted as meaningful and hence a candidate for truth. Truth begins with simple facts which reflect empirical reality; theories are then built up from these facts.

Karl Popper, attacking the positivist position, says no there are never enough separate facts to establish a scientific theory to be true. No matter how many facts that have been observed, we have no way of knowing whether the next observation will be contrary. To make the point in mathematical terms, I may establish that P(n) is true for all n up to 100 billion, yet that does not allow me to claim P(n) is true for all numbers. For example, P(n) could be a classic proposition such as the Goldbach conjecture, that every even number can be written as the sum of two prime numbers, which has been shown to be true for numbers as high as $4*10^{11}$, but is still not an established as a theorem for all even numbers.

Popper proposed instead the method of falsifiablilty. A statement is scientific if one can state the conditions which would falsify its claim. Science is thus the process of eliminating claims that are not true and a theory holds as long as there is not an experimental result that counters it.

Kuhn's thesis was that science was not a steady, cumulative acquisition of knowledge, but instead, periods of normal science interrupted by periodic intellectual revolutions. Kuhn was also responsible for popularizing the term paradigm, which he described as a collection of beliefs shared by scientists, a set of agreements about how problems are to be understood.

A significant variation on this theme can be found in the writings of Imre Lakatos who was educated as a mathematician and whose early work was in the philosophy of mathematics. Lakatos was bothered by the Kuhn's relativism and the lack of criteria for choosing between differing paradigms. He proposed instead the concept of a "Research Program" which consists of a hard-core theory, which is protected from falsification, surrounded by a protective belt of "auxiliary hypotheses." The auxiliary hypotheses are devised to protect the hard core, but they themselves are subject to falsification at which time the auxiliary hypotheses would be modified or replaced. Lakatos also proposed a method by which competing research programs could be evaluated. Research programs continually make modifications to explain new findings. If the modification preserves the content of its predecessor while explaining the new data and at the same time predicts new facts, some of which turn out to be true, then we say the research program is *progressive*. If, however, the program modification only accommodates the anomaly for which it was devised, but adds no new facts or discovery, then we call that research program *degenerative*. In this way we can see how new research programs can progress and eventually replace competing programs.[2]

In this manner Lakatos sought to avoid the relativism of Kuhn and others, while seeing the development of science as having a context and criteria for its development. To replace mathematics as the source of truth, modern science has taken up the claim to be the source of true knowledge. Yet science, progressing past the early 20th century positivism is now seen as a series of research programs, always changing and developing. New models are being developed, yet none can claim to have arrived at final truth.

A number of philosophers and theologians have learned from these developments in science as seen in the writings Nancey Murphy and Philip Clayton. They argue that while we cannot claim human absolute truth, we can build theological research programs similar to what Imre Lakatos proposed for science. This theological model would have its hard-core concepts and then auxiliary hypotheses that surround and protect the core. These auxiliary hypotheses would change as new evidence and new concepts develop either from religious experience, new understanding of scripture or from theology itself. Such programs can be seen to be progressive or degenerative depending on how much they can explain and how they deal with conflicting issues. As in science it is the community that in the final analysis decides the viability of the program.

In theology as well as in mathematics, we see ourselves building models, establishing paradigms, from which we can better view the world. Absolute truth is God's truth, and we are not God, nor can we confidently claim to know the "mind of God."

QUEST FOR FOUNDATIONS:

Descartes questioned and doubted everything, doubted his senses, doubted his tradition, questioned what the authorities had told him. This radical questioning finally led him to the realization that if everything was to be doubted, then he could not doubt, doubt itself. The thinking process that led to such doubt was the foundation for the certainty that he sought. Since he doubted, since he could think, then this was his foundation. *Cogito ergo sum*, "I think therefore I am", became the foundation from which he would reconstruct what was true and certain. Having a proper foundation upon which to build the rest of knowledge became the model for valid human knowledge. The metaphor of the building and its foundation was paramount. In Descartes' own words:

> "…individual owners often have their [houses] torn down and rebuilt, and even that they may be forced to do so when the building is crumbling with age, or when the foundations is not firm and it is in danger of collapsing. By this example I was convinced that… as far as the opinions which I had been receiving since my birth were concerned, I could not do better than reject them completely for once in my lifetime, and to resume them afterwards, or perhaps accept better ones in their place, when I had determined how they fitted into a rational scheme. And I firmly believed that by this means I would succeed in conducting my life much better than if I built only upon the old foundations and gave credence to the principles which I had acquired in my childhood without ever having examined them to see whether they were true or not." [Descartes 1637, as quoted in Murphy 1997, 10]

One of the major tenets of the modern view of knowledge is this epistemology. How do we know something is true? It is built carefully and rationally on a firm foundation. In a manner similar to the axioms of geometry, the self-evident truths are the bases, the foundations for developing a science, which is true, objective, universal, and timeless. All of this is built on the modern paradigm as the ideal for human knowledge, which is general, timeless, theoretical, objective, impersonal, universal and built on a firm foundation. And at the base, the foundation was mathematics.

As Kepler, Newton and others developed a mathematically based science, they were doing God's work. Mathematics was not just providing a convenient way to express physical principles; it was providing new methods of explanation. While some scientists objected to the lack of a mechanical explanation for such an action at a distance in the case of gravity, the mathematical formulas worked and were highly predictive. This "mathematization of science" was in a context in which scientists and mathematicians assumed they were discovering the truth by which God created the world and the natural laws which govern it. Kepler considered the laws of nature to be the mathematical thoughts of God and Newton expressed his work would lead to a belief in a Deity.

Yet as this mathematization continued the God-foundation receded into the background while the mathematics became more sophisticated and dominant. The deep religious convictions of the early scientists gave way to a more deistic assumption and to finally the skepticism of Hume and the dropping of the God-hypothesis completely as exemplified by the famous response of Laplace to Napoleon.

About this same time the discoveries of non-Euclidean geometries further brought question to the nature of mathematical truth. Mathematics pursued the establishment of foundations for its own discipline. If mathematics was no longer the structure for God's creation, then what was its basis? At the end of the 19th century and the beginning of the 20th there were a number of proposed answers to this question. The quest for foundations begun by Descartes became a major activity for mathematics.

For such a mathematical foundation the first problem encountered is the basic definitions of the terms. In Euclid's *Elements,* he attempted to define all of his terms. Point, for example was defined as "position without extension" and line as "length without breadth". How do you define very simple words by using more complex concepts?

For an everyday example, go to the Oxford Concise Dictionary for a definition of the word "dog." You will find this definition, among others:

dog n. 1. carnivorous quadruped of the genus Canis

Providing precise definitions are not as easy as we would like. In fact, the attempt to define words leads either to an infinite regression, or becomes circular. The way out for the abstract mathematical systems developed by Frege, Russell and others was to admit that we must begin with undefined terms. We find a similar logic problem when we try to prove the basic axioms. It is either an infinite regression or it becomes circular. So in such mathematical systems we begin with unproven axioms.

This led Bertrand Russell to declare that mathematics is the subject where we don't know what we are talking about (undefined terms), or whether what we are saying is true (the unproven axioms).

David Hilbert proposed a formalist foundation for mathematics which took as its goal establishing the consistency and completeness of the mathematical systems. If we can no longer claim absolute truth, let's at least try to show absolute consistency.

This program met a major crisis with the publication of Kurt Gödel famous paper "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems." [Gödel 1931] It was the goal of the formalist program to be able to prove within the system every meaningful statement as either true or false. Such a system was said to be complete. Gödel's theorem proved that in any mathematical system which includes the basic structure of arithmetic, there were statements that were in fact true, but could not be proven from within the axiom system established. Furthermore Gödel also went on to prove that such an axiom system cannot be shown to be free from contradictions. This feature of the mathematical foundation was a major result and is often compared to the theory of relativity and the Heisenberg Uncertainty Principle as one of the major developments of the 20th Century.

The foundations where cracking, in fact it seemed that there were no foundations. As Wittgenstein puts it: "One might almost say that these foundation-walls are carried by the whole house." [Wittgenstein 1969, 33]

As long as it was assumed that is was God's language and God's mind that was being discovered when we did mathematics and science, there was little need for further concern for foundations. Even consistency was not a major concern because it was assumed that the real world, especially God's created world would not contain inconsistency. As mathematics drifted further away from the assumption of discovering the laws of nature and as science left behind any sense of discovering God's world, the idea that mathematics was merely a human creation became dominant. What then was the nature of mathematical truth? If mathematicians were merely developing systems without concern for ultimate truth, then what was the foundation for mathematics?

Foundationalism was also being severely questioned in the philosophical and theological worlds as well. Nicholas Wolterstorff, among others, argued the collapse of classical foundationalism in his books: *Faith and Rationality*, [1983] and *Reason within the Bounds of Religion*, [1976].

Wentzel Van Huyssteen, Professor of Theology and Science at Princeton Theological Seminary, develops post-foundational theology in a series of books:

Essays in Postfoundationalist Theology, [1997],
Duet or Duel? Theology and Science in a Postmodern World, [1998] and
The Shaping of Rationality, [1999].

He argues that in science and in religion we relate to our world through interpreted experience in the context of a community and a specific tradition. There is no independent standing ground for human understanding or rationality. Our knowledge forms a groundless web of interrelated beliefs.

For van Huyssteen, post-foundationalism acknowledges the contextuality of both theology and science in human culture and affirms the crucial role of interpreted experience and the way tradition shapes our epistemic and non-epistemic values. "Instead of one perfect representation of God, or the world,…it may yield a 'collage' of knowledge that aims to be the most reliable, most useful, and most meaningful that we have." [Van Huyssteen 1998, 27]

Since Descartes we have sought to deny tradition, subjectivity, and context. We have during this Modern era sought certainty in firm foundations and in objective knowledge that denies the subjective and inner meaning. We are discovering anew that we all, even mathematicians and scientists, are parts of communities and traditions that provide us the context for our work and the languages for our theories. Our rationality is not the precision of a Euclidean geometry. In science we choose to look primarily at that which can be measured, repeated, experimented, and controlled. Yet there is much that falls outside our own methodological net. Even in mathematics where we have given up on the definition of basic terms, the self-evidence of our axioms, and even ultimate meaning of the mathematical system, we have not been able to affirm a foundation that can even yield absolute consistency.

QUEST FOR CERTAINTY:

In the Great Books, Syntopicon, the chapter on mathematics states:

> Mathematics has been honored for the precision of its concepts, the rigor of its demonstrations, the certitude of its truth. In all epochs mathematics has been looked upon as the type of certain and exact knowledge. It exemplifies rational truth, the spirit of dispassionate thought, the power of the human mind to rise above sensible particulars and contingent events to universal and necessary relationships. [Great Books, Syntopicon, Vol. 2, p. 42]

In her essay on "God, Truth, and Mathematics in Nineteenth Century England", Joan Richards describes the relationship between certainty in mathematics and in religion during the period following Newton. For Locke, a contemporary of Newton, the existence of God is "the most obvious truth that reason discovers, and though its evidence be…equal to mathematical certainty: yet it requires thought and attention; and the mind must apply itself to a regular deduction of it from some part of our intuitive knowledge." [As quoted in Richards 1992, 51] But following the development of non-Euclidean geometry in the early 19th century, Richards documents those who were fond of pointing out that it was no longer the indisputable case that the sum of the angles in a triangle would always be 180 degrees. "The major exemplar of necessary truth was itself not true, and the central epistemological structure within which English religious ideas had been interpreted began to crumble." [Richards 1991, 72-3]

In the 20th century, Bertrand Russell's study of logic, mathematics and philosophy began with a commitment to establish the certainty of mathematics:

> I wanted certainty in the kind of way in which people want religious faith. I thought that certainty is more likely to be found in mathematics than elsewhere…after some twenty years of arduous toil, I came to the conclusion that there was nothing more that I could do in the way of making mathematical knowledge indubitable" [Russell 1956]
>
> The splendid certainty which I had always hoped to find in mathematics was lost in a bewildering maze…It is truly a complicated conceptual labyrinth. [Russell 1959]

Science has had success by choosing to look only at the portion of reality that is measurable, repeatable, and controllable. Mathematics has refined that search by looking at the structure of argument and theory, eliminating also the empirical. Mathematics is not deterred, assisted or confused by empirical facts. Certainly with such a narrowing of the field, mathematics could find a firm foundation and certainty. Yet in spite of centuries of work and the best efforts of great minds, we now see that even in mathematics, consistency cannot be proved, completeness is beyond even simple systems that include basic arithmetic.

Our scientific and even mathematical search has led us to a loss of certainty, a distrust of foundations, and for some a complete relativism. From the religious perspective, however, we can see this as an acceptance of our creatureliness. We are not God and we do not have a God's eye view. We can, however, build models in our science, our mathematics and in our theology. These models assist us in understanding the physical world, the world of abstract reasoning and the world of relationship and meaning. But it also leads to an "epistemological humility" and respect for multiple perspectives. Mathematics has become richer by its ever-expanding concepts such as number, by its recognition that building models can help us understand reality, and that multiple perspectives can enrich both our understanding and our appreciation for our world, our neighbor, and our faith.

We are model builders, and lens grinders. We have become more aware of the lenses through which we view the world. There are many kinds of lenses, those for the near-sighted, those for the far-sighted, binoculars for mid range distance, telescopes for the heavens, microscopes for the very small. We are not postmodern relativists because we affirm a real world that is being viewed. But neither are we absolutists, we accept that it is not God's view that we enjoy. Humility is actually something that we should accept in our claim to understand absolute Truth.

Historically, both mathematics and theology have claimed the title: "Queen of the sciences." Both fields have a special relationship to science and its efforts to understand the creation. Each field has struggled with its own claim of absolute knowledge, yet both have learned that such claims have come under judgment and cannot be maintained. Perhaps each field can learn from the other and find new productive relationships to the sciences.

Recent authors have picked up this issue of certainty, both from a philosophical as well as a mathematical perspective. Mathematician, James Bradley, in the final chapter of *Mathematics in a Postmodern Age* [2004] has called for "epistemological humility" in both mathematics and theology. Philosopher, Brian Austin, building on the writings of C. S. Peirce, sees in the loss of certainty an exciting beginning of faith, an "exercise of volition," and a plan of action. [Austin 2000, 172]

Daniel Taylor in his book: *The Myth of Certainty: The Reflective Christian and the Risk of Commitment* sets forth our challenge:

> While certainty is beyond our reach, meaning—something far more valuable—is not. Meaning derives from a right relationship with God, based not on certainty and conformity, but on risk and commitment. [Taylor 1992, 94]
>
> We can see the human desire for certainty … for what it is—an understandable but ultimately doomed rebellion against human finitude. [121]

The lenses we use are helpful, valuable, and even necessary. While we use them to better see reality, we are also cognizant of the distortions that they may bring. Our epistemological humility requires that we affirm, as did the Apostle Paul in his letter to the Corinthians:

> "For now we see through a glass darkly; but then we shall see face to face."
> I Cor. 13:13

Jane Goodall [1990] has stated a similar concept in her book, *Through a Window*:

> There are many windows through which we can look out into the world, searching for meaning. Most of us peer through but one of these windows. And even that one is often misted over by the breath of our finite humanity.

NOTES

1. Among the notable exceptions see: Henry 1976, Howell & Bradley 2001, and Achtner 2004.
2. For introductions to Lakatos' philosophy of science and its relationship to theology see: Clayton 1989, Murphy 1990, and Murphy 1997.

REFERENCES

Achtner, Wolfgang. 2004. "Infinity in Science and Religion." Metanexus Conference, University of Pennsylvania, June 2004.

Austin, Brian. 2000. *The End of Certainty and the Beginning of Faith.* Macon, Georgia: Smyth & Helwys Publishing.

Bell, Eric Temple. 1934. *The Search for Truth.* Baltimore: Williams & Wilkins Co.

Clayton, Philip. 1989. *Explanation from Physics to Theology: An Essay in Rationality and Religion.* New Haven: Yale University Press.

Cohen, Morris R. 1918. "The Subject Matter of Formal Logic," *Journal of Philosophy, 15* (December 5, 1918).

Davis, Philip J. and Reuben Hersh. 1981. *The Mathematical Experience*. Boston: Houghton Mifflin Company.

Descartes, Rene. 1637. "Discourse on Method and Mediations." As quoted in Murphy 1997.

Gödel, Kurt. "On Formally Undecidable Propositions Of Principia Mathematica and Related Systems." Reprinted in J. van Heijenoort. 1967. *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*. Cambridge: Harvard University Press.

Goodall, Jane, *Through a Window: My Thirty Years with the Chimpanzees of Gombe.* Boston: Houghton Mifflin Co, 1990.

Guillen, Michael. 1983. *Bridges to Infinity: The Human Side of Mathematics.* Boston: Houghton Mifflin.

Henry, Granville C. 1976. *Logos: Mathematics and Christian Theology.* Lewisburg: Bucknell University Press.

Howell, Russell W., and W. James Bradley. 2004. *Mathematics in a Postmodern Age: A Christian Perspective.* Grand Rapids, Michigan: Eerdmans Publishing Company.

Kline, Morris. 1953. *Mathematics in Western Culture*. New York: Oxford University Press.

-------. 1980. *Mathematics: The Loss of Certainty*. New York: Oxford University Press.

Murphy, Nancey. 1990. *Theology in the Age of Scientific Reasoning.* Ithaca, New York: Cornell University Press.

-------. 1997. *Anglo-American Postmodernity: Philosophical Perspectives on Science, Religion, and Ethics.* Boulder, CO: Westview Press.

Edward Purcell. 1973. *The Crisis of Democratic Theory: Scientific Naturalism & the Problem of Value.* Lexington: University Press of Kentucky.

Richards, Joan L. 1992. "God, Truth, and Mathematics in Nineteenth Century England", pages 51-78 in *The Invention of Physical Science: Intersections of Mathematics, Theology and Natural Philosophy Since the Seventeenth Century.* Edited by Mary Jo Nye, Joan L. Richards, and Roger H. Stuewer. Kluwer Academic Publishers.

Russell, Bertrand. 1945. *History of Western Philosophy.* New York: Simon and Schuster

-------. 1956. *Portraits from Memory.* New York: Simon and Schuster.

-------. 1959. *My Philosophical Development.* New York: Simon and Schuster

Taylor, Daniel. 1992. *The Myth of Certainty: The Reflective Christian and the Risk of Commitment.* Grand Rapids: Zondervan Publishing House.

Van Huyssteen, Wentzel. 1997. *Essays in Postfoundationalism Theology.* Grand Rapids, Michigan: Eerdmans Publishing Company.

-------. 1998. *Duet or Duel? Theology and Science in a Postmodern World*. London: SCM Press

-------. 1999. *The Shaping of Rationality*. Grand Rapids, Michigan: Eerdmans Publishing Company.

Wittgenstein, Ludwig. 1969. *On Certainty.* New York: Harper & Row.

Wolterstorff, Nicholas. 1976. *Reason within the Bounds of Religion*. Grand Rapids, Michigan: Eerdmans Publishing Company.

-------. With Alvin Plantinga. 1984. *Faith and Rationality.* Notre Dame: Notre Dame Press.